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Scientific Paper No. 24

An Atmospheric Diffusion Slide-rule

by C. E. WALLINGTON, M.Sc.

LONDON: HER MAJESTY'S STATIONERY OFFICE

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An Atmospheric Diffusion Slide-rule

by C. E. Wallington, M.Sc.

SUMMARY

The paper describes a slide-rule that can be used to calculate concentrations and dosages* in clouds of aerosols being transported and diffused by atmospheric wind and turbulence.

The slide-rule includes scales for incorporating into the calculations several methods of assessing depths and widths of diffusing clouds, but the relative merits of the methods are not discussed in detail; the main purpose of the paper is to present the slide-rule as a calculating aid.

The slide-rule is not intended for laymen to the subject of atmospheric diffusion; it is more for those who have at least a little understanding of the theoretical background. For such users the slide-rule provides a means of predicting or assessing experimental diffusion observations; it facilitates comparison of various methods of diffusion calculations and it enables a user to compile tables or graphs suitable for use by laymen.

1. INTRODUCTION

Atmospheric diffusion calculations are usually complicated, and take a long time to carry out without special computing aids. The slide-rule described in this paper is such an aid. It contains facilities for rapid calculation of concentrations or dosages of airborne material in four types of diffusing clouds whose depth and width dimensions can be specified or estimated.

Three methods of estimating the depth dimension of a cloud and four methods of estimating the width of a cloud are included in the slide-rule operations, but it is stressed that this paper is concerned with speeding up calculations and not so much with the applicability or justification of the various methods.

In methods described in this paper as *wind fluctuation* and *wind shear* methods the distinction between instantaneous and continuous sources can be automatically allowed for to some extent in the data used for the calculations, but in the method described as the *diffusion category* method the distinction is ignored in order to provide a simple method which can be used as a rough guide when only the bare minimum of meteorological data is available. As a justification for this approach it can be argued that any method which yields rapidly calculated concentrations or dosages to within a factor of about three of the real answer is a useful goal in atmospheric diffusion problems.

The slide-rule is designed for a wide range of users, from those who have to base their calculations on a glance at the sky and an assessment of the wind to those who may have wind fluctuation data and upper wind measurements for their basic data. Facilities for using a diffusion-coefficient method are also included mainly to allow an experimenter to assess relationships between wind fluctuation and wind shear parameters and diffusion coefficients.

The calculations are assumed to be for diffusion over fairly flat uniform terrain in practically homogeneous atmospheric diffusion conditions. No account is taken of effects of such phenomena as sea breezes, mountain and valley winds, lee waves, thunderstorm downdraughts, or marked meteorological changes over the range for which a calculation is made. Here again it is stressed that the slide-rule is a calculating aid for applying certain methods; it is not a full guide to the applicability of the methods in particular circumstances.

* Dosage in this paper denotes the integral of concentration with time over an infinite time interval.

2. THE SLIDE-RULE

Figure 1 and Plate I illustrate the layout of the slide-rule. It has two slides which will be referred to as the upper and lower slides, and two cursor plates which carry several cursor lines and scales.

Scales on the frame and slides (Figure 1(*a*)) are numbered (1 to 20) at their left-hand ends, the numbering being from left to right and from top to bottom. Cursor lines and scales on the cursors (Figure 1(*b*) and (*c*)) are lettered (circled A to Q), double letters (circled AA to JJ) are used to denote setting marks on the frame and slides, and here, too, the sequence is from left to right and from top to bottom. There are three additional marks, "d_2", "d_1" and "shear" on the lower slide. The names of the scales, and the units where necessary, are listed at the ends of the rule.

When the slide-rule operations are described, the scales, cursors and marks will be referred to by their numbers or letters.

The letters, H to Q, marked at points on vertical lines on the upper slide and the cursor plates do not refer to cursors or scales; they denote diffusion categories which will be used in one of the computing methods.

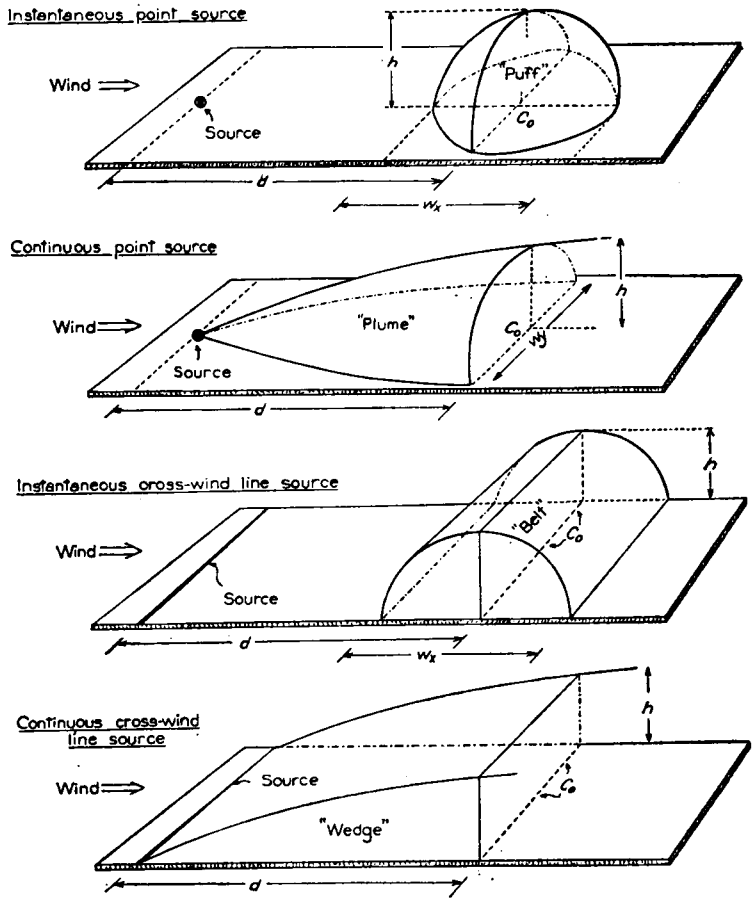


FIGURE 2. Assumed forms of diffusing clouds from four types of ground-level sources. At a distance d from the source the "depth" of each cloud cross-section is denoted by the symbol h, while the "widths" are denoted by w_x for the axis lying in the direction of the mean wind and by w_y for the cross-wind direction. The convention used to specify these "depth" and "width" dimensions is shown in Figure 3. C_0 is the peak concentration.

On the back of the slide-rule (Figure 1(*d*)) there is a summary of the slide-rule operations required for various types of calculation. The large number of operations may appear formidable at first sight, but normally only a few are required for a calculation. (See foldout for Figure 1.)

3. DIFFUSING CLOUD FORMS

Four types of sources are mentioned in the top section of the slide-rule. Each type produces a distinctive form of diffusing cloud—as illustrated in Figure 2.

In this paper the names "puff", "plume", "belt" and "wedge" will be used to describe the diffusing clouds from an instantaneous point source, a continuous point source, an instantaneous cross-wind line source and a continuous cross-wind line source respectively. Experiments have shown that, for many practical purposes, the distribution of concentration of the diffusing material along the "depth" and "width" axes illustrated in Figure 2 can be taken as Gaussian—as shown in Figure 3. This assumption is used for the slide-rule calculations, together with the convention

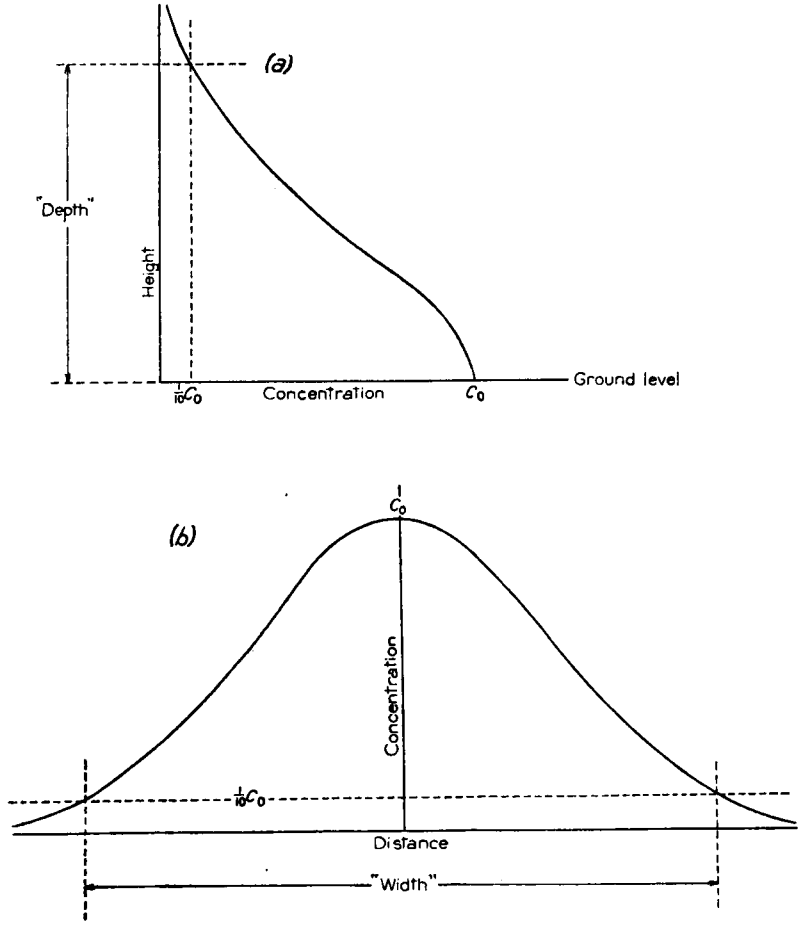


FIGURE 3. Graphs to illustrate the assumed Gaussian distribution of concentration along the "depth" and "width" axes of cloud cross-sections.

The convention used is that the concentration at the boundaries used to denote "depth" and "width" is one tenth of that of the peak concentration C_0.

(*a*) Graph of concentration versus height along the depth axis of a diffusing cloud.

(*b*) Graph of concentration versus distance along the width axis of a diffusing cloud.

that the concentration at the boundaries used to denote "depth" and "width" of a cloud cross-section is one tenth of the peak concentration marked as C_0 in Figures 2 and 3.

It can be shown* that, if the wind speed, u, is constant with height, the relationships between the quantity of material, Q, emitted at a source and the peak concentrations, C_0, are as follows:

Peak concentration in a puff

If Q grams of material are emitted at an instant at a point at ground level, then the peak concentration, C_0, at ground level, when the depth and widths of the puff are h, w_x and w_y metres, will be given by

$$C_0=(1{\cdot}716)^3\, Q/hw_xw_y \quad \text{grams/cubic metre} . \qquad \ldots\ldots \quad (1)$$

Peak concentration in a plume

If Q gm/second of material are emitted continuously from a point at ground level in a wind of u m/sec, then the peak concentration, C_0, at ground level, where the depth and width of the plume are h and w_y metres will be given by

$$C_0=(1{\cdot}716)^2\, Q/uhw_y \quad \text{gm/m}^3 . \qquad \ldots\ldots \quad (2)$$

Peak concentration in a belt

If material is emitted at an instant uniformly along a long cross-wind line, the quantity being Q gm per metre length of line, then the peak concentration, C_0, on the axis of the belt at ground level, when the depth and width of the belt are h and w_x metres, will be given by

$$C_0=(1{\cdot}716)^2\, Q/hw_x \text{ gm/m}^3 . \qquad \ldots\ldots \quad (3)$$

Peak concentration in a wedge

If material is emitted continuously and uniformly along a long cross-wind line in a wind of u m/sec, the rate being Q gm/sec per metre length of line, then the peak concentration, C_0, at ground level, where the depth of the wedge is h metres, will be given by

$$C_0=1{\cdot}716\, Q/uh \text{ gm/m}^3 . \qquad \ldots\ldots \quad (4)$$

The dosage of material at a point is conventionally defined as the integral $\int C\mathrm{d}t$, where C denotes concentration, t denotes time, and the integral is taken over whatever time is specified. Total dosages possible are usually obtained by integrating over an infinitely long time interval. The total dosage, D_0, received at a point in the path of the centre of a puff is given approximately by the equation

$$D_0=(1{\cdot}716)^2\, Q/uhw_y \text{ gm sec/m}^3 , \qquad \ldots\ldots \quad (5)$$

where Q, u, h and w_y are as already indicated above, the h and w_y dimensions being the depth and cross-wind width of the puff as it passes the point.

The total dosage received from a belt as it passes a ground-level point is given approximately by

$$D_0=1{\cdot}716\, Q/uh \text{ gm sec/m}^3 , \qquad \ldots\ldots \quad (6)$$

where h is the depth of the belt as it passes the point.

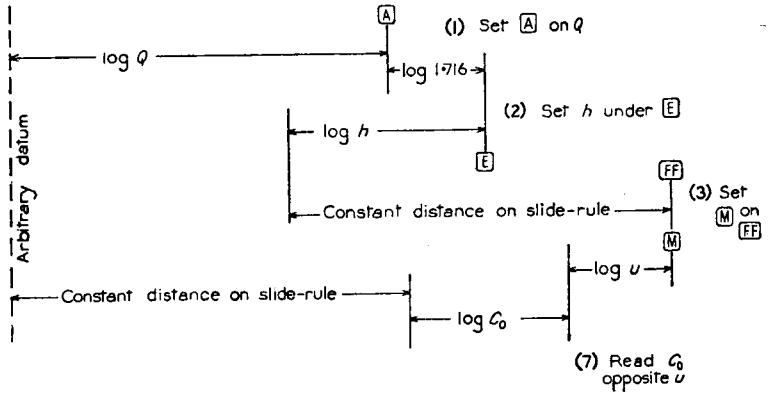
* The equations used are simple cases of those included in *Atmospheric diffusion.*[1]†

† The superscript figures refer to the bibliography on page 24.

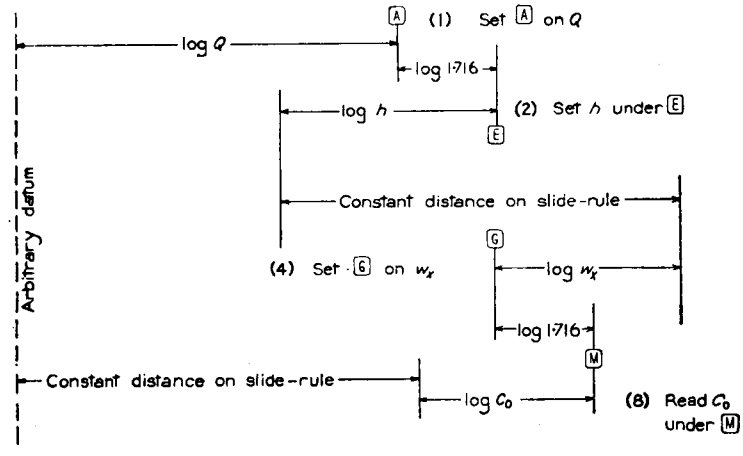
4. CALCULATION OF PEAK CONCENTRATIONS AND DOSAGES

Before making a calculation the source strength, Q, and wind speed, u, are normally known or estimated. If the depth and width dimensions of a cross-section of the cloud can be determined the peak concentrations and dosages can be calculated. Methods of estimating these depth and width dimensions will be described in later sections of this paper. To illustrate the basic slide-rule operations, let us assume, for the present, that depths and widtlis are known.

Table I indicates the slide-rule operations required for the calculations mentioned so far, and Figure 4 illustrates the principle relating the belt- and wedge-type operations to equations (3), (4) and (6).



(*a*) Wedge calculation— $C_0=1{\cdot}716Q/uh$.



(*b*) Belt calculation— $C_0=(1{\cdot}716)^2Q/hw_x$.

FIGURE 4. The principles relating wedge- and belt-type slide-rule operations to the peak concentration formulae.
The numbers in brackets refer to slide-rule operations described in more detail in Table I, and the circled letters denote cursors or marks on the slide-rule.
Dosage calculations for a belt involve exactly the same operations as those used for concentrations in a wedge.

TABLE I. *Basic slide-rule operations*

Operations required for the various types of calculation are indicated by the numbers. SLIDE-RULE OPERATIONS	TYPE OF CALCULATION: Puff C_0 $\frac{(1\cdot716)^3Q}{hw_xw_y}$	Puff D_0 $\frac{(1\cdot716)^2Q}{uhw_y}$	Plume C_0 $\frac{(1\cdot716)^2Q}{uhw_y}$	Belt C_0 $\frac{(1\cdot716)^2Q}{hw_x}$	Belt D_0 $\frac{(1\cdot716)\,Q}{uh}$	Wedge C_0 $\frac{(1\cdot716)\,Q}{uh}$
Set cursor line A on source strength Q either in units of pounds, metres and seconds on scale 1 or in units of grams, metres and seconds on scale 2.	1		1	1		1
Move upper slide so that the depth of cloud, h, on scale 4 is under cursor line E.	2		2	2		2
Set cursor line M on mark FF.						3
Set cursor line G on along-wind width, w_x, on scale 5.	4			4		
Move upper slide so that the mark FF is aligned with cursor line M.	5					
Set cursor line G on cross-wind width, w_y, on scale 5.	6		6			
Read D_0 or C_0, now on scale 10 opposite value of speed of travel, u, on scale L.			7			7
Read C_0, now under cursor line M on scale 10.	8			8		

5. OFF-PEAK CALCULATIONS

The concentration, C_0, at ground level under the axis of a belt of cloud from an instantaneous cross-wind line source emitted at a height h_s, is related to the peak concentration on the axis of the belt, C_A, by the formula

$$C_0 = C_A \exp\left[-\tfrac{1}{2}\,(2\cdot15h_s/h)^2\right] \qquad \ldots\ldots\ (7)$$

which may be rewritten as

$$\log_e C_0 = \log_e C_A - 2\cdot311 \exp\left[2\log_e (h_s/h)\right] \qquad \ldots\ldots\ (8)$$

h being the depth of the belt above the source level, as shown in Figure 5.

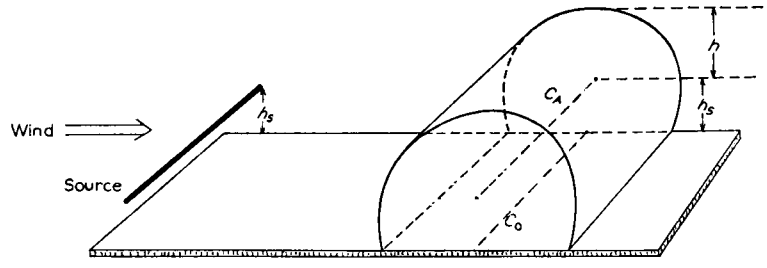


FIGURE 5. Cloud form produced by an elevated instantaneous cross-wind line source.
The height of the source is denoted by h_s and the depth used in the slide-rule calculations by h. C_A and C_0 denote the peak concentrations on the axis of the belt and at ground level respectively.

Translation of this equation into slide-rule operations is as follows:

Suppose a belt type of calculation has just been carried out, leaving the cursor line M over the peak concentration (on the axis of the belt) on scale 10 and the cursor line E over the depth, h, on scale 4. To find the concentration at ground level, take note of the "adjustment number" on scale D opposite the value of h_s on scale 4. Then move the "adjustment scale", 13, so that the adjustment number noted is under the cursor line M. The required concentration will now be indicated on scale 11 by the mark GG at the left-hand end of scale 13. Figure 6 shows the principle of these operations.

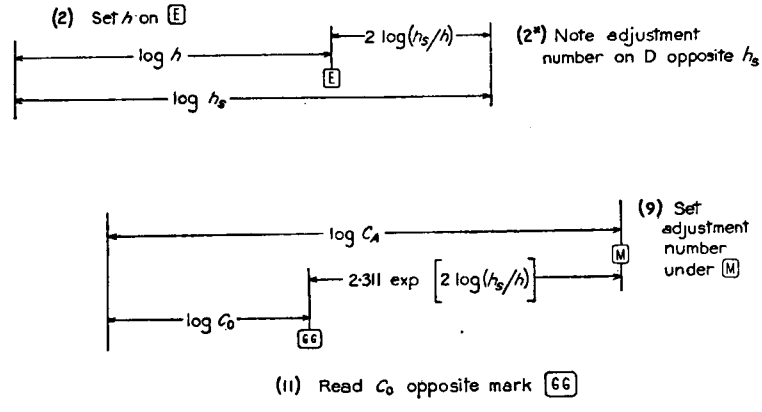


FIGURE 6. Principle of slide-rule operations to calculate peak ground-level concentrations in a cloud from an elevated source.
The notation is the same as in Figures 4 and 5. The numbers actually marked on the adjustment scales in the slide-rule do not denote actual values of 2 $\log_e$ (h_s/h); they are linear functions of this quantity on a convenient scale of 0 to 15.
Adjustment for height of source is $\log_e C_0 = \log_e C_A - 2{\cdot}311 \exp [2\log(h_s/h)]$.

Off-peak concentrations, C, or dosages, D, at various along-wind and cross-wind distances from the centre or axis of a cloud can be calculated in a similar manner using scale J for adjustment numbers appropriate to horizontal distances. Table II lists the operations, which are an extension

TABLE II. *Extension of Table I to cover off-peak calculations*

Basic operations 1–8 are described in Table I. The extended operations enable concentrations, C, and dosages, D, to be calculated for points off the centre or axis of a cloud.		TYPE OF CALCULATION					
		Puff		Plume	Belt		Wedge
		C	D	C	C	D	C
Basic operations	SLIDE-RULE OPERATIONS	1		1	1		1
	As in Table I.	2		2	2		2
	Note adjustment number on scale D opposite height of source, h_s, on scale 4.	2*		2*	2*		2*
							3
	As in Table I.	4			4		
	Note adjustment number on scale J opposite value of x on scale 5.	4*			4*		
	As in Table I.	5					
		6		6			
	Note adjustment number on scale J opposite value of y on scale 5.	6*		6*			
				7			7
	As in Table I.	8			8		
Adjustment for height of source	Move scale 13 so that on this scale the adjustment number noted in 2* is under cursor line M.	9		9	9		9
	Read C_0 or D_0 now on scale 11 opposite value of speed of travel, u, on scale 12.			10			10
	Read C_0 now on scale 11 opposite mark GG.	11			11		
Adjustment for x	Move scale 13 so that on this scale the adjustment number noted in 4* is under cursor line M.	12			12		
	Read C_x now on scale 11 opposite mark GG.	13			13		
Adjustment for y	Move scale 13 so that on this scale the adjustment number noted in 6* is under cursor line M.	14		14			
	Read C_y or D_y now on scale 11 opposite value of speed of travel u, on scale 12.			15			
	Read C_y now on scale 11 opposite mark GG.	16					

of those described in Table I, and Figure 7 illustrates the notation being used to locate and refer to off-peak positions and values.

The calculation of concentrations or dosages at a height z in a cloud from a ground-level source can be performed in the same way as that for ground-level concentrations or dosages in an elevated cloud, the value z being used in place of h_s in the operations in Table II.

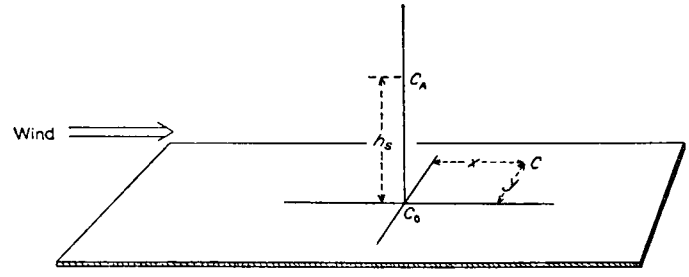
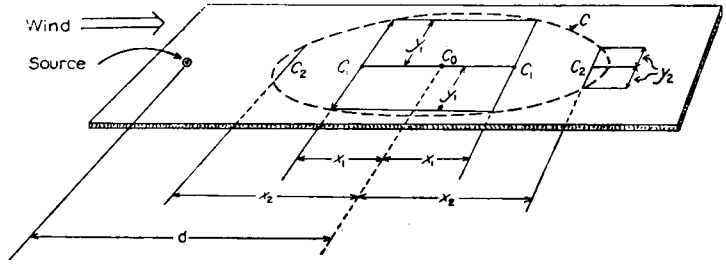
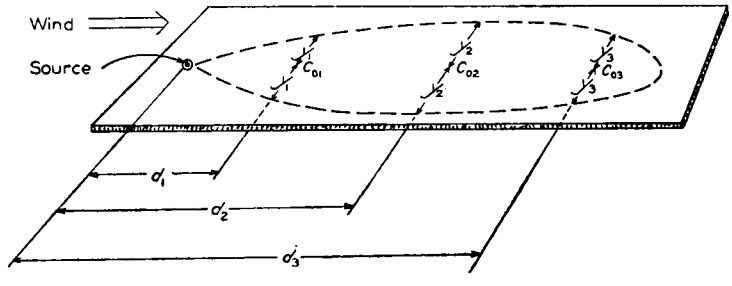


FIGURE 7. Notation used to refer to off-peak positions and concentrations.
C_A denotes the concentration on the axis of the cloud at the height, h_s, of the source, C_0 is the concentration at ground level beneath the position of C_A, and C is the ground-level concentration at a point (x,y) relative to the position of C_0.

The adjustment-number system can also be used to locate points on the boundaries of areas in which concentrations or dosages are greater than a given value. Table III lists the operations required to deal with the various types of sources and Figure 8 illustrates the descriptive notation of the calculations and the method of dealing with a puff calculation is elaborated in the caption.



(*a*) Concentration isopleth in a puff.



(*b*) Concentration isopleth in a plume.

FIGURE 8. Diagrams to illustrate the technique for determining isopleths of given concentrations.
In the puff type of calculation the peak ground-level concentration, C_0, is calculated for the given distance d. An adjustment is then made to compute concentrations, C_1, at an arbitrary distance, x_1, from the peak. Table III operations are then used to calculate the cross-wind distance, y_1, at which the required concentration, C, is located. The operations are repeated for several along-wind distances, x_2, x_3, etc., until enough points are obtained to draw the isopleth of given concentration.

TABLE III. *Operations for calculating points on boundaries of areas in which concentrations or dosages are greater than given values C or D*

		Puff		Plume	Belt		Wedge
		C	*D*	*C*	*C*	*D*	*C*
Basic operations		As listed in Tables I and II					
Adjustment operations, if necessary.	If adjustment operation, 9, 12 or 14 has just been carried out, move cursor M to mark GG.	17		17	17		
Co-ordinate calculations	Move lower slide so that given value of *C* or *D* on scale 11 is opposite speed of travel, *u*, on scale 12.			18			
	Move lower slide so that given value of *C* on scale 11 is opposite mark GG.	19			19		
	Note adjustment number on scale 13 under cursor line M.	20		20	20		
	Move cursor line G to width, w_x or w_y, on scale 5 (if not already there).	21		21	21		
	Read magnitude of co-ordinate, $\pm x$ or $\pm y$, of points (where the concentration or dosage is equal to the given value *C* or *D*) now on scale 5 opposite the value of the adjustment number (noted in 20) on scale J.	22		22	22		

6. DURATION OF A PASSAGE OF A PUFF OR BELT

In this paper the duration of passage of a puff or belt over a point will be taken as the time taken for the width of the cloud to cross the point, in other words the width divided by the speed of travel, *u*. The slide-rule includes a scale to perform this division. If the cursor line G is set on the width on scale 5, the duration of passage can be read on scale 8 opposite the speed of travel on scale K.

The duration for which the concentration at a point affected by a puff or belt exceeds a given value may be obtained by first calculating the along-wind distance, *x*, from the centre of the puff or axis of the belt where the given concentration occurs. The method of calculation has just been described in Section 2 and Table III. The required duration is approximately twice this distance divided by the speed of travel, *u*; so if cursor line G is set on $2x$ on scale 5, the duration can be read on scale 8 opposite the speed of travel on scale K. The slide-rule operations for duration calculations are numbered 23 and 24 in the complete table of instructions on the back of the slide-rule (see Figure 1(*d*)).

7. ESTIMATION OF THE DEPTH OF A CLOUD

Three methods of estimating the depth, h, can be used with the slide-rule. They are described below:

(i) *The diffusion category method.*—Pasquill[2] suggested that relationships between the depth, h, of a cloud and distance, d, of travel from the source can be categorized according to wind speed and sky conditions, for example in light winds and strong sunshine the depth of a cloud increases rapidly with distance from the source, whereas on cloudless nights with light winds the pronounced low-level stability usually produced by night cooling tends to inhibit vertical diffusion. The method can be applied to the slide-rule calculations with the aid of the diffusion category chart shown in Figure 9, and the group of curves under scale 4 on the left-hand half of the upper slide. These curves will be called the "(h, d) lines". The sections of the (h, d) lines yielding depth of more than 3 km are tentative extrapolations from Pasquill's estimates.

To find the depth of a diffusing cloud at a distance d from its source, the state of sky should be observed and the wind speed measured or estimated according to the guidance on the chart. These two items are used to determine the diffusion category in the alphabetical range H to Q. The diffusion category letter is then used in the operations described under 2*a* in Table IV.

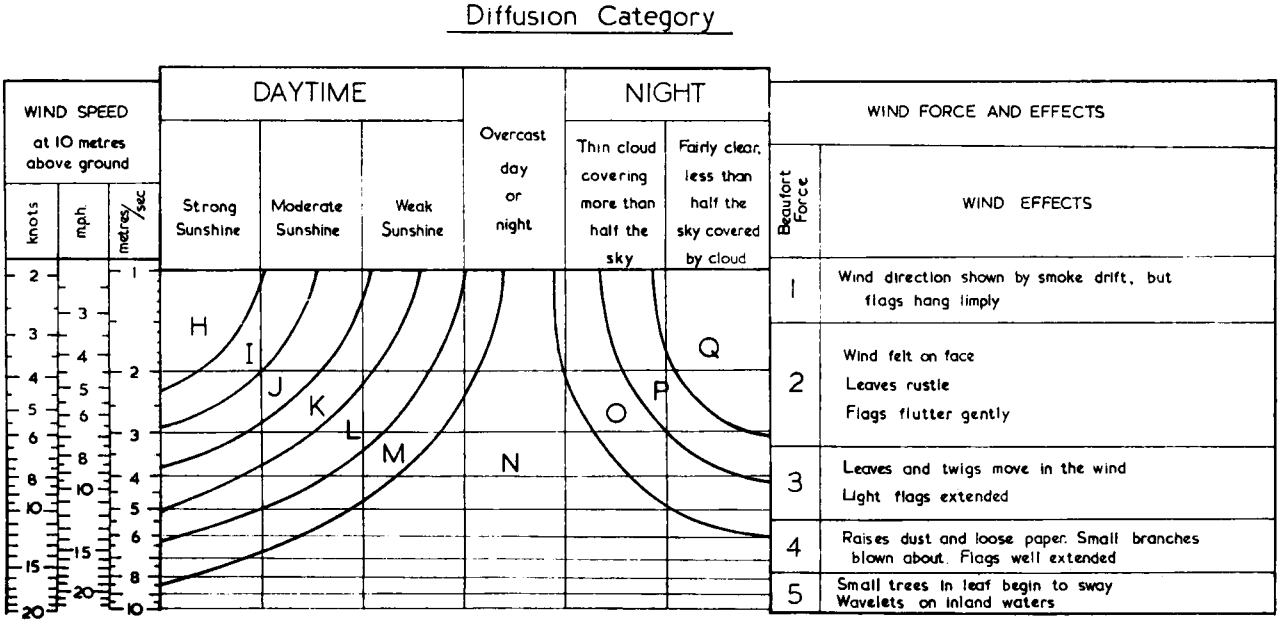


FIGURE 9. Diffusion category chart.

The diffusion category is designated by the letter corresponding to the observed state of the sky and the measured or estimated wind speed at 10 metres above open ground.

(ii) *A wind fluctuation method.*—A technique evolved by Hay and Pasquill[3] relates the depth, h, of a cloud cross-section to the standard deviation of the wind inclination by the equation

$$h \simeq 2{\cdot}15 d\sigma_\alpha \qquad \ldots\ldots (9)$$

where d is the distance from the source and σ_α is the standard deviation of running averages of wind inclination, the averaging time being a specified fraction of the time taken for the material in the

TABLE IV. *Methods of setting the "depth" parameter*

	DIFFUSION CATEGORY METHOD	WIND FLUCTUATION METHOD	DIFFUSION COEFFICIENT METHOD
Data required	State of sky observation and wind speed at 10 m above open ground.	Standard deviation, σ_α, of wind inclination.	Vertical diffusion coefficient, and speed or time of travel of cloud.
	2a	*2b*	*2c*
Operations equivalent to operation 2 in Table I	(i) Use data to determine diffusion category letter from the chart. (ii) Move upper slide so that distance, *d*, on the (*h*, *d*) lines under scale 4 intersects cursor line E at the diffusion category letter.	(i) Set mark AA on source strength on scale 2. (ii) Move left-hand cursor to set σ_α, in degrees on scale B or in radians on scale C, on the mark BB. (iii) Move the upper slide to set distance, *d*, on scale 3 under cursor line E.	(i) Note: At the start of the calculations the source strength must be taken as 1 on scale 2. (ii) Set cursor line F on time of travel, *t*, on scale 20 *or* set mark d_1 on distance, *d*, on scale 17, then set cursor line F on speed of travel, *u*, on scale 14. (iii) Move upper slide to set diffusion coefficient, K_z, on scale 9 under cursor line E.
Extra operations required to determine value of depth, *h*		(iv) Move cursor line A to source strength on scales 1 or 2.	(iv) Move cursor line A to 1 on scale 2.
	2e		
	Read value of depth, *h*, now on scale 4 under cursor line E.		
Operation to introduce the effect of a "lid" on vertical diffusion	*2f*		
	Check that the value on scale 4 under cursor line A does not exceed the height, *H*, of the "lid" on vertical diffusion. If it does exceed this value, set *H* on scale 4 under cursor line A. The value on scale 4 under cursor A will be referred to as *h'*.		

cross-section to travel from the source. The average value of the fraction deduced from early experiments was a quarter, and a subsequent study by Thompson[4] suggests that this fraction may range from 1/15 in very stable conditions to about 1 in unstable conditions. However, the main purpose here is not to discuss the assessment of the fraction, but to describe how a given value of σ_α can be incorporated into the slide-rule calculations. The equation

$$h \simeq 2{\cdot}15d \tan \sigma_\alpha \qquad \ldots\ldots (10)$$

is used as a slightly more precise version of the depth and distance formula, and the slide-rule operations are listed under *2b* in Table IV.

(iii) *The diffusion coefficient method.*—One of the oldest empirical methods of estimating the depth, h, is the use of the equation*

$$h=2{\cdot}15\sqrt{(2K_z\, d/u)} \qquad \ldots\ldots \quad (12)$$

or

$$h=2{\cdot}15\sqrt{(2K_z\, t)} \qquad \ldots\ldots \quad (13)$$

where t is the time taken for the diffusing material to travel from the source to the cross-section, and K_z is a coefficient whose value is given. The design of the slide-rule is such that when this method is used the source strength must initially be taken as 1 on scale 2. This is not a serious drawback as the multiplication by the source strength, Q, in all the concentration and dosage formulae can be carried out later. Operations for the diffusion coefficient method are listed under 2*c* in Table IV.

8. ALLOWANCE FOR A "LID" ON VERTICAL DIFFUSION

The height to which material can be spread upwards by vertical diffusion is normally limited. On cloudless nights with light winds there is often a temperature inversion or stable layer at very low levels. Anticyclonic conditions often produce a stable layer whose base is usually between about 1000 m and 3000 m above MSL, and even when deep convection stirs up the air to great heights the tropopause, at about 10 to 15 km in temperate latitudes, sets the upper limit to which this stirring can occur. Vigorous convection currents can burst through a stable layer and convection and turbulence can destroy low-level inversions. Nevertheless, for most calculations it can be assumed that a "layer" of pronounced stability imposes a "lid" on vertical diffusion. The height, H, of this lid above ground level cannot be estimated by the methods described in Section 7; it is an additional item of data that is required for medium- and long-range diffusion calculations. Meteorologists can estimate the height of a lid with the aid of temperature soundings aloft and forecasting techniques appropriate to the particular atmospheric and topographical conditions involved. A low-level lid can sometimes be seen to limit the rise of smoke plumes or the vertical extent of smoke haze, and aircraft pilots can sometimes detect a lid by noting a sharp contrast between the turbulence below and the relatively smooth flight conditions above.

To see how allowance can be made for the lid effect, consider diffusion in a wedge of material from a continuous cross-wind line source. Figure 10 shows calculated concentration/height diagrams for three distances from the source. The full lines are the Gaussian distributions which would be expected if the lid did not exist, and the bold pecked lines are distributions calculated by a method (not yet described) which takes the lid into account.

Up to the distance where h is less than the height of the lid, H, the effect of the lid is unimportant. Beyond this distance, the simple slide-rule calculations yield concentrations which are too low. At ground level, however, this error is small, at least up to the distance where $h=1{\cdot}716H$. From this distance (which will be denoted by d_1) onwards, better accuracy is attained by assuming the material to be spread out uniformly with height through the layer of depth H. This assumption is equivalent to limiting h to $1{\cdot}716H$ in the slide-rule calculations—which means that the value on scale 4 under cursor line A must not exceed H. This operation, under the label 2*f* in Table III, completes the depth setting operations which apply to all of the four types of sources, and not merely to the wedge type used in the above argument.

* See *Atmospheric diffusion*,[1] page 77.

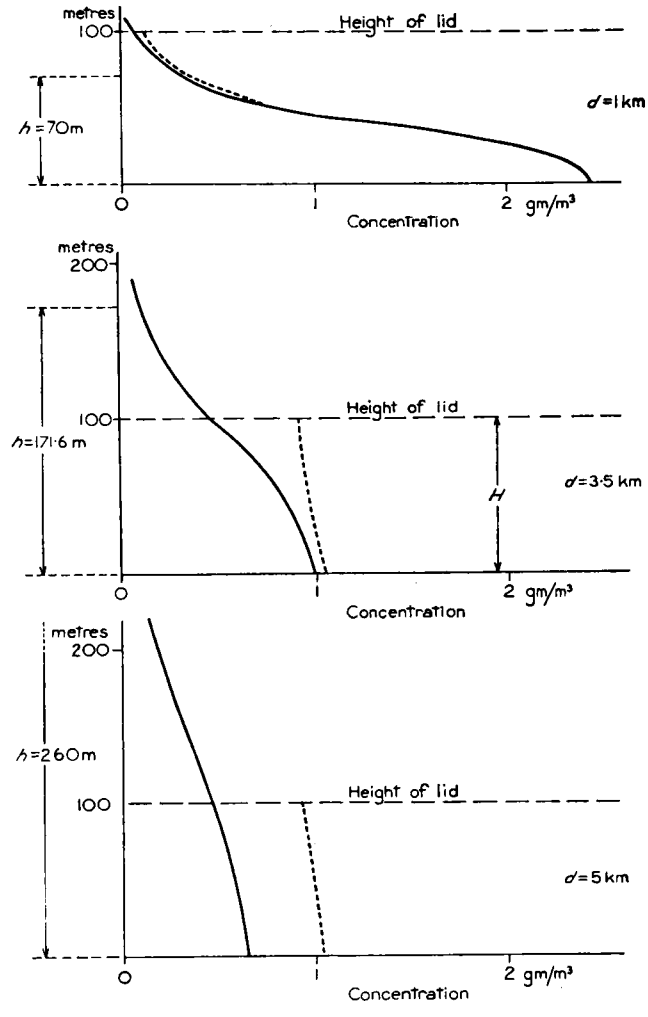


FIGURE 10. Graphs of concentration versus height in a wedge for three distances from a continuous cross-wind line source at ground level.

——— Gaussian distributions expected if there were no lid on vertical diffusions.

– – – – Distributions calculated by a method which takes into account a lid at 100 metres above ground level.

For this example the source is of strength 100 gm per sec per metre, the speed of travel is 1 metre/sec and the depths, h, are obtained by the diffusion category method using category N. Up to the distance where $h<1{\cdot}716H$ the effect of the lid can be ignored when computing ground-level concentrations, but beyond this distance an assumption of a vertically uniform concentration, of 1 gm/m^3, is approximately correct, whereas neglect of the lid produces considerable errors.

When a lid is effective, the depth, h, as defined in this paper no longer signifies depth in the common sense of the word.

The value of a vertically uniform concentration is easily calculated to be

$$C=Q/uH \qquad \ldots\ldots \quad (14)$$

9. ESTIMATION OF THE WIDTH OF A CLOUD

The slide-rule has facilities for four methods of estimating the width of a cloud cross-section. The first three of the methods described below are of the same type as the three depth estimation methods.

(i) *The diffusion category method.*—Wind and sky conditions are used to determine the diffusion category on the chart in Figure 9, and the curves under scale 5 on the right-hand half of the upper slide are used in the operations listed under 4*a* and 6*a* in Table V. The curves will be called the "(*w*,*d*) lines". The broken lines for distances less than 100 m and greater than 100 km are tentative extrapolations from Pasquill's estimates.

Strictly the (*w*,*d*) lines refer to clouds from sources at which material is emitted for periods of a few minutes only. Even so, over a wide range of atmospheric and operational conditions use of these lines will probably yield estimated widths to within a factor of three for the four types of sources dealt within this paper.

(ii) *The wind fluctuation method.*—This method postulates that the width can be calculated using any of the equations

$$w_y \simeq 4{\cdot}3d \tan \sigma_\theta, \quad \ldots\ldots \quad (15)$$

$$w_y \simeq 2d \tan \tfrac{1}{2}\theta_w, \quad \ldots\ldots \quad (16)$$

$$w_x \simeq 4{\cdot}3\sigma_u\, d/u, \quad \ldots\ldots \quad (17)$$

$$w_y \simeq 4{\cdot}3\sigma_v\, d/u, \quad \ldots\ldots \quad (18)$$

where σ_θ is the standard deviation of the wind direction
θ_w is the range over which the wind direction fluctuates
σ_u is the standard deviation of wind speed along the x axis
σ_v is the standard deviation of wind speed in the y (cross-wind) direction

} the averaging time for measurements of these parameters is a specified fraction of the time the material takes to travel from the source to the cross-section.

d is the distance from the source.
u is the speed of travel of the cloud.

The mean value of the fraction relating averaging time to time of travel appears to be about $\frac{1}{4}$; however, the main purpose here is not to discuss assessments of the fraction, but to describe how given values of wind fluctuation measurements can be incorporated with the slide-rule calculations. The calculating procedures are listed under 4*b* and 6*b* in Table V.

Scale H for setting σ_θ or θ_w is a hybrid in that it is correct for equation (15) up to 30° and for equation (16) for values greater than 30°. But the overall accuracy is adequate because θ_w seldom exceeds 30° and if θ_w is less than 30° values of $2d \tan \frac{1}{2}\theta_w$ and $4{\cdot}3d \tan (\theta_w/4{\cdot}3)$ differ by only small amounts.

TABLE V. *Methods of setting width parameters*

	DIFFUSION CATEGORY METHOD	WIND FLUCTUATION METHOD	DIFFUSION COEFFICIENT METHOD	WIND SHEAR METHOD
Data required	State of sky observation and wind speed at 10 m above open ground.	*For* w_x: Standard deviation, σ_u, of wind in x direction and speed of travel of cloud, *For* w_y: Standard deviation, σ_v, of wind in y direction and speed of travel of cloud. *or* standard deviation, σ_θ, of wind direction. *or* Range, θ_w, over which wind direction fluctuates.	Speed or time of travel of cloud *and* horizontal diffusion coefficient, K_x, for w_x *and* K_y for w_y.	Speed of travel of cloud *and* wind speed components up to a height h', noted in $2f$, u components being required for w_x, v components for w_y.
	4*a*	4*b*	4*c*	4*d*
Operations equivalent to operation 4 in Table I	(i) Use data to determine diffusion category letter from the chart (Figure 9).	(i) Move right-hand cursor so that σ_u on scale K is set on mark EE.	(i) Move right-hand cursor to set cursor line K either on time, t, on scale 6, or on distance, d, on scale 7.	*If* $d \leqslant d_2$: (i) From wind data obtain wind shear, u, over the depth h', whose value was noted in $2f$.
	(ii) Move right-hand cursor so that distance, d, on the (w, d) lines intersects cursor line G at the diffusion category letter.	(ii) Move the lower slide so that distance, d, on scale 16 is set under cursor line Q.	(ii) Move lower slide to set K_x on scale 15 under cursor line K.	(ii) Set u on scale L on mark FF.
		(iii) Move right-hand cursor to set u on scale N on mark HH.	(iii) Move right-hand cursor so that, if time of travel, t, is given, cursor line Q is on mark JJ, or, if speed of travel, u_s is given, the value of u on scale P is on mark JJ.	(iii) Set d on scale 15 under cursor Q. (iv) Set Δu on scale O on mark II. *If* $d > d_2$: Apply the diffusion coefficient operations 4*c* with virtual diffusion coefficient K_u.

TABLE V—*continued*

Read width if required	4*e* Read value of width, w_x, now on scale 5 under cursor line G.			
Operations equivalent to operation 6 in Table I	6*a* Same as operations 4*a*.	6*b* *If σ_v is given*: Similar to operations 4*b* with σ_v in place of σ_u. *If σ_θ or θ_w is given*: (i) Set σ_θ in degrees on scale H or in radians on scale I over mark CC, *or* set θ_w in degrees on scale H over mark DD. (ii) Set d on scale 16 under cursor line Q. (iii) Move right-hand cursor to set cursor line Q on mark HH.	6*c* Similar to operations 4*c* with K_y in place of K_x.	6*d* Similar to operations 4*d* with Δv in place of Δu, and K_v in place of K_u.
Read width if required	6*e* Read value of width, w_y, now on scale 5 under cursor line G.			

(iii) *The diffusion coefficient method.*—This method uses the equation

$$w_y = 4{\cdot}3\sqrt{(2K_y\, d/u)} \quad \ldots\ldots \quad (19)$$

or

$$w_y = 4{\cdot}3\sqrt{(2K_y\, t)} \quad \ldots\ldots \quad (20)$$

where K_y is a given diffusion coefficient for the y (cross-wind) direction. Width, w_x, in the x direction can be computed on a similar basis by substituting a given along-wind diffusion coefficient, K_x, for K_y in the above equations. The slide-rule operations to perform these calculations are described under 4*c* and 6*c* in Table V.

(iv) *A wind shear method.*—Tyldesley and Wallington[5] have suggested that horizontal dispersion over all but very short ranges is controlled mainly by wind shear and vertical diffusion. The dispersion mechanism can be described as follows: vertical diffusion spreads the material out in depth; differential advection due to wind shear causes the material to cover a larger horizontal area, and over this area vertical diffusion continues to spread the material out in depth.

Horizontal dispersion by this mechanism appears to go through three distinct phases, namely:

(1) A phase in which the cloud is not high enough to be affected by whatever lid there may be on the vertical extent of diffusion.

(2) A transition phase wherein the effect of the lid begins to reduce the rate of horizontal dispersion.

(3) A final phase in which the material is spread out almost uniformly with height and in which the height of the lid is an important factor in determining the rate of horizontal dispersion.

In the first phase the width of a cloud cross-section can be calculated approximately with the equation

$$w_y \simeq 0{\cdot}48d\frac{\Delta v}{u} \qquad \ldots\ldots (21)$$

where Δv is the shear in the cross-wind speeds, v, over a height $h/1{\cdot}716$, h being the depth of the cloud as estimated by any of the methods described in Section 7. The value $h/1{\cdot}716$ will be denoted by h', and it will be seen later that the value of this h' can be easily determined during the course of normal slide-rule operations.

The value of h' becomes equal to the height of the lid, H, at a distance d_1 (introduced in Section 8) and at this point Δv becomes the difference between cross-wind speeds at ground level and at the level of the lid. Tyldesley and Wallington consider the first phase to end at the distance d_1.

For distances beyond d_1 the depth of cloud is kept constant in accordance with the procedure for making allowance for the lid. Thus h' remains equal to H, Δv remains constant and the width increases linearly with distance. This can be considered as the transition phase, which numerical studies suggest continues to a distance of about $4{\cdot}2d_1$, which will be denoted by d_2. Operations for determining d_2 are included in Table VI.

TABLE VI. *Determination of d_2 (distance beyond which the lid on vertical diffusion becomes important in the wind shear effect) and virtual diffusion coefficients K_u and K_v*

I	Carry out operations 2*a*, *b* or *c* for increasing values of distance of travel, *d*, and note the distance at which the value on scale 4 under cursor line A just becomes equal to the height of lid, *H*. Denote this distance by d_1.
II	Move the lower slide to set the mark d_1 on the value of d_1 on scale 17.
III	Read the value of d_2 now on scale 17 opposite mark d_2.
IV	Move the left-hand cursor so that the cursor line F is over the speed of travel, *u*, on scale 14.
V	Move the lower slide to set the value of Δu (the difference between the x component of the wind at ground level and that at height H) on scale 14 under cursor F.
VI	Read the value of K_u now on scale 15 opposite the value of Δu on scale 18.

The value of K_v, for the cross-wind virtual diffusion, can be determined by carrying out the above operations with Δv in place of Δu.

In the final phase, which we take to be from d_2 onwards, the width can be calculated approximately by the equations

$$w_y \simeq 4{\cdot}3\sqrt{(2K_v\, d/u)} \qquad \ldots\ldots (22)$$

or

$$w_y \simeq 4{\cdot}3\sqrt{(2K_v\, t)} \qquad \ldots\ldots (23)$$

where $K_v = 0{\cdot}026\,(\Delta v)^2\, d_1/u$, Δv is wind shear over height H, and K_v is a virtual diffusion coefficient used to describe the eventual wind shear effect.

The slide-rule operations required to put this method into effect are described under 4*d* and 6*d* in Table V and the operations in Table VI may be required to determine d_2 and calculate K_y. It can be noted that the first two phases are both covered by one set of operations; the change from the first to the second phase is automatically effected by making allowance for the lid in the depth setting operations. Width, w_x, in the x direction may be computed on a similar basis by substituting along-wind parameters, K_x, K_u and Δu for K_y, K_v and Δv in the equations.

10. SHEAR IN DIFFUSING CLOUDS

Wind shear not only contributes to horizontal dispersion; it also produces a shear in diffusing clouds. Figure 11 illustrates the effect of wind shear on a belt from an instantaneous cross-wind line source.

In this illustration, the increase of wind with height moves material in the upper part of the belt on faster than that at lower levels, but when an upper lid on vertical diffusion becomes effective the rate of shearing is reduced and eventually reaches a limit. At this limit, the shearing action of the wind is balanced by vertical diffusion, as indicated by the bold horizontal and vertical arrows depicting a circulation within the shaded area in the cross-section in Figure 11.

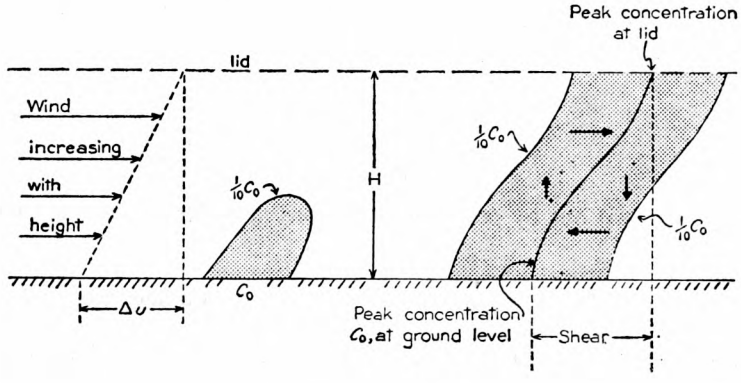


FIGURE 11. Shearing action in a belt of cloud. Cross-sections at two stages in the progress of the belt are shown.

In the first stage, on the left, the lid imposes little or no control on the shearing section. In a later phase, on the right, when the peak concentration at any level is almost uniform with height, the effect of the lid is to limit the shearing action. At this stage differential horizontal advection within the cloud, indicated by the horizontal bold arrows within the shaded area, is balanced by differential vertical diffusion, indicated by the vertical bold arrows.

In this paper the limiting shear is specified as the distance between the positions of the peak concentrations at ground level and at the lid. It can be shown that if the wind shear is constant with height then the value, S, of this limiting shear is given approximately by

$$S = 0{\cdot}26\ \Delta u\ d_1/u \qquad \ldots\ldots\ (24)$$

where Δu, u and d_1 are the wind shear, mean wind speed and distance respectively, as already defined in Sections 8 and 9.

Slide-rule operations for calculating S are exactly the same as I to V in Table VI; these operations leave the arrow labelled "shear" pointing to the value of S on scale 17. More detailed study of diffusion computations has suggested that this limiting value of the shear is almost reached by the time the cloud has travelled a distance of $2d_1$ from the source. The slide-rule does not include a facility for calculating the shear in a cloud at shorter ranges, but as the value of the shear is seldom more than about one tenth of the distance of travel, knowledge of the approximate limiting value is sufficient for most practical purposes.

The effect of cross-wind shear in plumes can be calculated in the same way, using Δv in place of Δu, and as a guide to assessing value of shear and position of a plume, Figure 12 shows typical paths traced out by the peak concentrations at the top and bottom of two plumes.

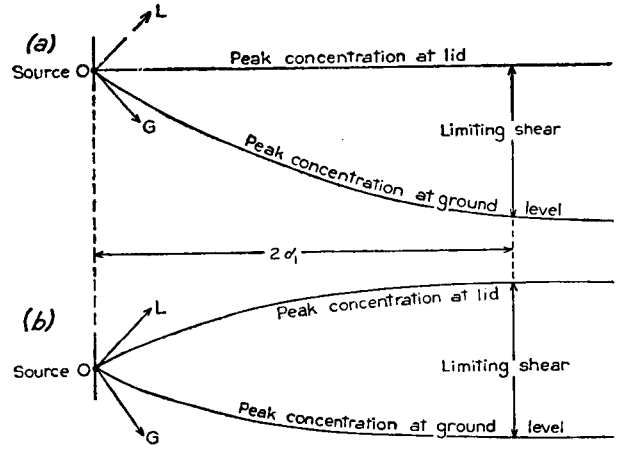


FIGURE 12. Typical paths traced out by peak concentrations at ground level and at the upper lid in two plumes.

(*a*) Plume from ground-level source.

(*b*) Plume from source half way between ground level and lid, i.e. at a height of $H/2$.

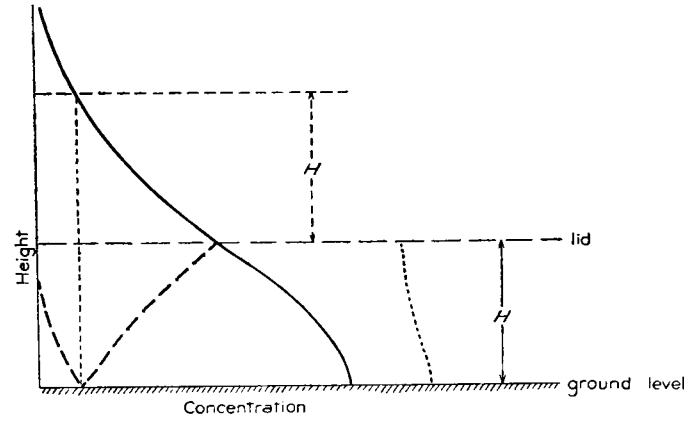
The limiting shear is the same in both cases.

OL wind at lid OG wind at ground level d_1 is defined on page 18.

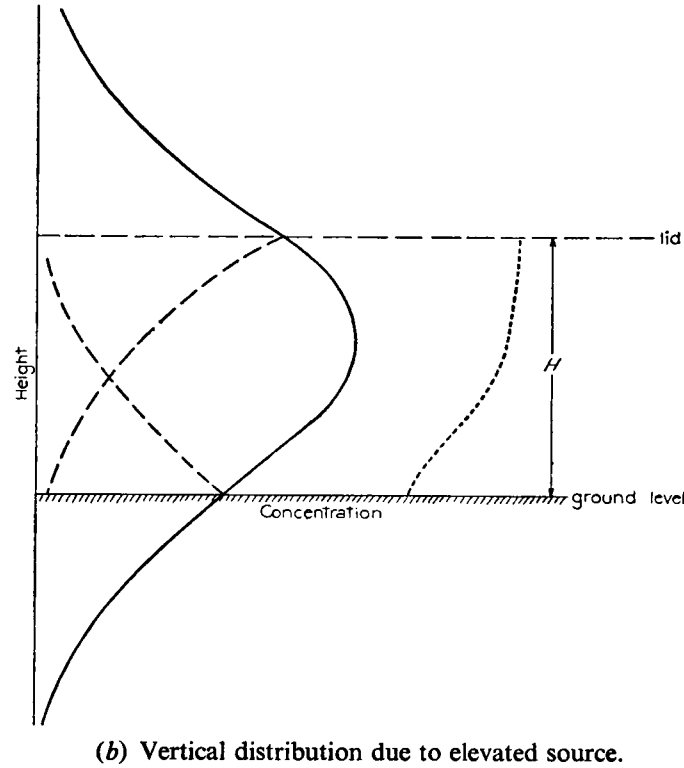
11. MORE ACCURATE ALLOWANCE FOR THE EFFECT OF A LID

When the height of a source or the height for which a concentration is to be calculated is more than about one third of the height of the lid, the calculating methods described so far may produce appreciable errors. Figure 10 illustrates discrepancies between the simple slide-rule calculations and more accurate calculations of concentrations in a wedge. The method of making these more accurate calculations is based on the assumption that ground level and the lid can be treated as reflectors of the material being diffused towards them. This can be visualized as taking the assumed Gaussian distribution to be folded back on itself at the ground and at the lid—as depicted in Figure 13(*a*). A more accurate graph of the concentration at any height in the diffusion régime is then the sum of the curves between the lid and the ground in Figure 13(*a*). Thus the accuracy of the slide-rule calculations can be improved either by the illustrated graphical technique, or by summing values calculated for several height adjustment values; for example, concentration at a height z due to a ground-level source with a lid at height H, is the sum of concentrations calculated by the simple system for heights z, $2H-z$, $2H+z$, . . ., etc. In practice, no more than two heights would be required because contributions from greater heights would be small. A similar technique can be used to calculate concentrations or dosages at a given height in clouds from elevated sources, but it should be noted that the effect of ground-level reflection on ground-level concentrations or

dosages is already included in the formulae stated in equations (1) to (8). This is because most requirements will be for ground-level calculations. Therefore, if the more accurate extension of the calculating method involves adding contributions from curves initially folded back at ground level, the sum of the concentrations should be divided by two. Figure 13(*b*) illustrates the initial folding back at ground level.



(*a*) Vertical distribution due to ground-level source.



(*b*) Vertical distribution due to elevated source.

FIGURE 13. The distribution of material with height between ground level and a lid.
This can be visualized as a Gaussian distribution folded back on itself at ground level and at the lid.
——— Simple Gaussian distribution.
— — — Gaussian distribution "folded" into diffusion régime between ground level and lid.
- - - Sum of curves in diffusion régime; in (*b*) this equals twice the values of required concentrations.

12. ESTIMATION OF SPEED AND DIRECTION OF TRAVEL OF A CLOUD

Close to a source the peak concentration near ground level tends to move with the mean wind velocity within a few metres of the ground, but as the cloud becomes deeper the wind velocities at higher levels contribute to the movement and, for many practical purposes, it is then adequate and convenient to assume the mean velocity of the cloud to be the mean wind velocity up to the height h', determined by operation $2f$ in Table IV. This height, h', is the same height as that used to determine the wind shear effect on the cloud. Ultimately, when the total amount of material at each level becomes uniform from ground level up to the height of the lid, the cloud as a whole moves with the mean wind velocity in the layer from ground level up to the lid.

If wind measurements up to the lid are not available, it is helpful to note that the wind at about 2000 feet is often veered by between 5° and 25° from the 10-metre wind, and its speed at 2000 feet is often about $1\frac{1}{2}$ times that at 10 m. But this guidance should be used with caution; in light winds or in very stable conditions the wind structure can vary considerably from one weather situation to another.

13. MOVING POINT SOURCES

When a trail of material is emitted from an aircraft flying at high speed across an airflow of relatively low speed, the source can be considered as an instantaneous cross-wind line source. But a trail from a slow-moving source, such as a land vehicle or a ship, is better treated as a moving plume. Using Figure 14 to illustrate the argument, suppose that we wish to calculate the concentration at point X

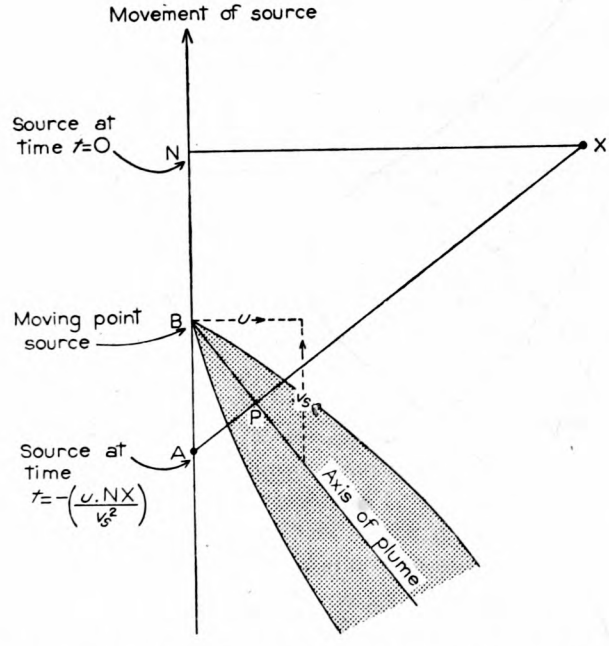


FIGURE 14. Diagram to illustrate the method of calculation of concentration at a sampling point x in a cloud of material emitted continuously from a point source moving at speed v_s across an airflow of mean velocity u in the diffusion régime.

$$BP=\left[NX+\frac{v_s^2}{u}t\right]\left[\frac{u}{\sqrt{(u^2+v_s^2)}}\right] \qquad PX=(NX-ut)\left[\frac{v_s}{\sqrt{(u^2+v_s^2)}}\right]$$

due to a continuous point source moving at speed v_s along the line ABN, which is perpendicular to the mean velocity u which would normally be the speed of travel of a cloud. The material will form a plume whose axis lies from the source at any instant along the direction of the vector difference between u and v_s—as shown in the diagram. When the source is at point B, the concentration at X can be calculated as an off-peak concentration at a distance PX from the axis of the plume for a distance of travel BP from a point source.

If a series of concentrations for various times is required, it is convenient to take the time $t=0$ when the source is at a point, N, directly upwind of the sampling point. Then at a time t,

$$\text{distance along the plume axis, BP} = \left(\text{NX} + \frac{v_s^2}{u}t\right) \frac{u}{\sqrt{(u^2+v_s^2)}} \qquad \ldots\ldots \quad (25)$$

and

$$\text{distance from the plume axis, PX} = \left(\text{NX} - ut\right) \frac{v_s}{\sqrt{(u^2+v_s^2)}}. \qquad \ldots\ldots \quad (26)$$

It is apparent from Figure 14 and equation (25) that this treatment assumes that the emission commences at or before the moving point reaches point A, i.e. at or before a time $t=-u.\text{NX}/v_s^2$.

14. CALCULATION OF CONCENTRATION-VERSUS-TIME GRAPHS

To compute concentrations at various times during the passage of, say, a belt of cloud, the procedure is to make off-peak calculations for various distances of travel of the cloud from the source. For example, at a time t after the emission from an instantaneous line source, the concentration at a distance x, downwind of the source, can be calculated as the concentration at a distance $|x-ut|$ from the axis of a belt whose distance of travel is ut, u being the speed of travel. The concentration-versus-time graph will not normally be Gaussian; it will be asymmetrical, the rate of increase in concentration as the belt approaches the sampling point being greater than the rate of decrease after the axis of the belt has passed. The graph will allow the duration of passage to be determined with slightly better accuracy than that obtained from the procedure described in Section 6.

15. MINIMUM LENGTH OF CROSS-WIND LINE SOURCES

Cross-wind line sources are not normally of infinite length, but the effects of an infinite line source can be obtained from a long finite line source provided it is long enough. The minimum length and position for a given distance of travel to a specified point can be estimated as follows. Calculate the cross-wind width of a plume in the same weather conditions for the given distance of travel; any one of the methods already described can be used. The required minimum length of line source can then be taken as this width plus whatever range of error may be assessed for prediction of the mean wind direction. The line source should be laid so that the mid-point of the belt is predicted to pass over the point.

16. THE EFFECT OF BARRIERS ON CROSS-WIND SPREAD

Suppose material is emitted at a continuous point source in a steep-sided valley on a cold clear night and drifts down the valley with a katabatic wind. In such a situation there is likely to be a lid on vertical diffusion at some fairly low level, and the valley sides may also limit the cross-wind spread of material. The slide-rule method of dealing with the lid effect has already been described in Section 8. A somewhat similar method can be applied to the effect of barriers to cross-wind diffusion. The required operation is listed as 6*f* in the complete table of operations shown in Figure 1(*d*).

This method could also be applied to, say, the spread of a puff travelling with the wind in a narrow road flanked by terraced houses or other continuous buildings, but it should be remembered that the assumption made in the calculations is that the barriers and lid are continuous boundaries. Furthermore, the meteorological data for open countryside would not necessarily apply to airflow and diffusion in rugged terrain or in built-up areas.

17. REVERSE OPERATIONS

In many atmospheric diffusion problems the question posed is: at what distance from a source will a given concentration or dosage occur ? These problems can be tackled with the slide-rule by trial and error processes; calculations are made for various trial distances until the required concentration or dosage is obtained. For some types of sources and methods the process is easy, for example when dealing with a belt type of problem by the diffusion category method, the appropriate cursor lines (A and M) are set on the source strength and given concentration and the upper slide is moved until the distances on the (*h*,*d*) lines and (*w*,*d*) lines under the diffusion category letters on cursors E and G are equal.

18. SUITABILITY OF THE METHODS

The curves marked on the slide-rule for the diffusion category method may well need to be modified in the light of future experience. But substitution of new curves for old on the slide-rule will be fairly easy, and the present curves can at least provide a plausible foundation on which to build refinements. The weakness of the diffusion category methods concerns estimation of widths rather than depths of cloud. But the depth estimation by this method can be used with moderate confidence.

The wind fluctuation method is mainly for use by meteorologists who have wind data available from either routine or experimental records. Many forecasters will have sufficient data to use the diffusion category method for depth estimation and the wind fluctuation method for width estimation, the value of θ_w being obtained from autographic records; this combination of methods is suitable and adequate for many practical purposes.

The diffusion coefficient method is included mainly for the experimenter who may wish to make calculations in terms of the "*K*" theory. But as there is often considerable doubt over the interpretation of the diffusion coefficients, considerable caution should be exercised when using this method.

Being the most recent of the methods described, the wind shear method has not yet been subjected to a large number of tests, but the method shows considerable promise and is appropriate to medium- and long-range problems.

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APPENDIX—EXAMPLES OF SLIDE-RULE CALCULATIONS

Slide-rule operations in the following examples are referred to by the numbers in the complete tables of operations on the back of the slide-rule (shown in Figure 1(*d*)).

Example 1

Source data: Material is emitted from a continuous point source at ground level at a steady rate of 100 gm/sec.

Meteorological data: Wind at 10 m is estimated to be 5 m/sec.
Sunshine is moderate.
Cumulus convection cloud is present at a height of at least 800 m.

Problem: What will be the concentration on the axis of the plume at ground level 2 km from the source ?

Solution:

Operation	*Remarks*
1	Set cursor A on 10^2 on scale 2.
2*a*(i)	From the diffusion category chart, a wind of 5 m/sec and moderate sunshine put the diffusion into category M.
2*a*(ii)	Set the 2-km line under point M on cursor E.
2*f*	The value of h' on scale 4 under cursor A is about 100 m. The fact that cumulus cloud is present at a height of at least 800 m means that if there is a lid present it will be at least as high as 800 m and can, therefore, be ignored in the present problem.
6*a*(ii)	Set point M on cursor G on the 2-km line of the (*w*,*d*) lines.
7	The required concentration can now be read on scale 10 opposite the speed of travel on scale L. This speed of travel can be taken as the mean speed up to height h', noted in 2*f* as 100 m. In this shallow layer of 100 m the mean wind speed is likely to be only slightly more than the 10-m wind speed. So 5 m/sec can be taken as the speed of travel. The concentration on scale 10 opposite 5 m/sec on scale L is 6×10^{-4} gm/m^3.

Example 2

Source data: 100 gm of material is emitted at an instant from a ground-level point source.

Meteorological data: As in Example 1.

Problem: What will be the dosage and peak concentration of a ground-level point 2 km downwind of the source ?

Solution:

Operation	*Remarks*
1 2*a*(i) 2*a*(ii) 2*f*	As in Example 1.
6*a*(ii)	Set point M on cursor G on the 2-km line of the (*w*,*d*) lines.
7	The required dosage can now be read on scale 10 opposite the speed of travel, taken to be 5 m/sec on scale L. This dosage is 6×10^{-4} gm sec/m^3.
5	Move mark FF to cursor M.
4*a*(ii)	Set point M on cursor G on the 2-km (*w*,*d*) line. Note that operations 4, 5 and 6 can be carried out in reverse order. Operation 4 performs the division by w_x, while 6 performs division by w_y. In the diffusion category method as applied to this slide-rule, w_x is taken to be equal to w_y.
8	Read the required peak concentration on scale 10 under cursor line M. This is 9×10^{-6} gm/m^3.

Example 3

Source data: Material is emitted at a rate of 5 gm/sec from a continuous ground-level point source.

Meteorological data: Slight haze with weak sunshine. Wind speed is 2 m/sec and it is reported that a temperature inversion is imposing a lid at 3000 m on vertical diffusion, and that the wind velocity is almost constant up to 3000 m.

Problem: Plot the isopleth of ground-level concentration of 6×10^{-6} gm/m^3.

Solution:

Operation	*Remarks*
1	Set cursor A on 5 on scale 2.
2*a*(i)	Diffusion category letter is found to be L.
18	Set 2 m/sec on scale 12 opposite the required concentration of 6×10^{-6} gm/m^3 on scale 11. This operation can be carried out now because no further operation requires the lower slide to be moved.
2*a*(ii)	Take 100 m as the first distance, i.e. set the 100-m line under point L on cursor E.
2*f*	Check for lid effect.
6*a*(ii)	Set point L on cursor G on the 100-m (*w*,*d*) lines.
20	Note adjustment number of approximately 12·5 on scale 13 under cursor M.
22	Read co-ordinate, $\pm y$, on scale 5 opposite 12·5 on scale J. This is about 45 m.

Repeat operations 2*a*(ii), 2*f*, 6*a*(ii), 20 and 22 for increasing values of distance of travel. The following table of results has been calculated.

Distance	$\pm y$ (approx.)	Remarks
100 m	45 m	
300 m	110 m	
500 m	150 m	
1 km	230 m	The value of h' under cursor A attains the height of the lid, 300 m, at a distance of just over 4 km. Therefore, for the calculations for 4 km and more the upper slide is left such that the 300 m on scale 4 is under cursor A.
2 km	320 m	
3 km	360 m	
4 km	320 m	
5 km	300 m	
6 km	200 m	
7 km	110 m	

These co-ordinates are used to plot the isopleth in Figure 15. Note that the isopleth is not drawn exactly through all of the calculated co-ordinates. This is because some of the numbers in the calculations can only be read or set approximately on the slide-rule.

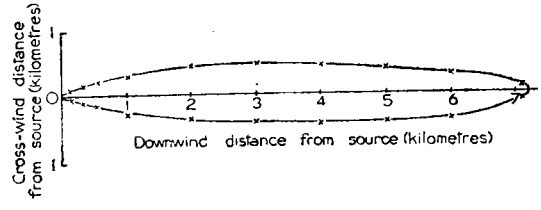

FIGURE 15. Plot of the concentration isopleth required in Example 3.
O Continuous point source emitting 5 gm/sec

The isopleth of ground-level concentration is of 6×10^{-6}gm/m^3; wind speed is 2 m/sec; there is a lid at 300 m and the diffusion category is L. Crosses denote the co-ordinate values calculated with the slide-rule.

Example 4

Source data: A practically instantaneous cross-wind line source is laid by an aircraft flying at 50 m above ground level, emitting material at a steady rate of 0·1 gm/sec m.

Meteorological data: Wind 3 m/sec, almost constant up to at least several hundred metres above ground. Overcast with cloud base higher than 700 m.

Problem: Plot a graph of ground-level dosage against downwind distance from the source.

Solution:

Operation	*Remarks*
1	Set cursor A on 10^{-1} on scale 2.
2*a*(i)	Diffusion category is N.
2*a*(ii)	Take 200 m as first distance.
2*	This distance yields an adjustment number of 14·5 for operation 2*. For distances less than about 160 m the adjustment number would be off scale D.
2*f*	Assume that the lid is at least as high as the low cloud base, i.e. at least 700 m.
3	Set cursor M on mark FF.
9	Set 14·5 on scale 13 under cursor M.
10	Read dosage at 200 m on scale 11 opposite wind speed, 3 m/sec, on scale 12. This yields $3{\cdot}8\times10^{-11}$ gm sec/m^3.

Repeat operations 2*a*(ii), 2*, 2*f*, 3, 9 and 10 for increasing values of distance of travel. The following table of results has been calculated.

Distance	Dosage in gm sec/m^3
200 m	$3{\cdot}8\times10^{-11}$
400 m	$8{\cdot}0\times10^{-6}$
600 m	$1{\cdot}2\times10^{-4}$
800 m	$2{\cdot}2\times10^{-4}$
1 km	$2{\cdot}7\times10^{-4}$
1·5 km	$3{\cdot}2\times10^{-4}$
2 km	$3{\cdot}2\times10^{-4}$
3 km	$2{\cdot}9\times10^{-4}$
4 km	$2{\cdot}5\times10^{-4}$
5 km	$2{\cdot}0\times10^{-4}$
10 km	$1{\cdot}4\times10^{-4}$

These values are used to obtain the required graph shown in Figure 16. Note that a logarithm scale is used for the dosage axis.

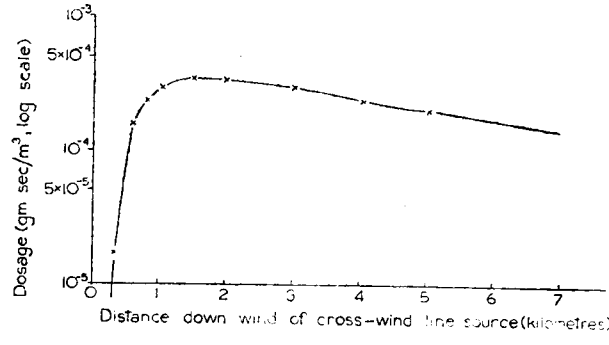

FIGURE 16. Plot of dosage versus distance required in Example 4.

Crosses denote the ground-level dosages from an instantaneous cross-wind line source of strength 0·1 gm/sec m, calculated for various distances. Wind speed is 3 m/sec; the height of the lid is at least 700 m, and the diffusion category is N.

Example 5

Source data: 1 kg of material is emitted at an instant at ground level.

Meteorological data: Wind 5 m/sec. Night-time with only a few stars visible through breaks in the clouds. There is no evidence of a lid below about 100 m.

Problem: Calculate the peak ground-level concentration in the puff of material as it passes a point 200 m downwind of the source.

Solution:

Operation	*Remarks*
1	Set cursor A on 10^3 on scale 2.
2*a*(i)	Diffusion category N.
2*a*(ii)	Set the 200-m (*h*,*d*) line under point N on cursor E.
2*f*	Whatever lid there may be is ineffective if it is above 100 m.
4*a*(ii)	Set point N on cursor G on 200 m on (*w*,*d*) lines.
5	Move upper slide so that mark FF is under cursor M.
6*a*(ii)	Set point N on cursor G on 200 m on (*w*,*d*) lines.
8	Read peak concentration at 200 m under cursor M on scale 10. This is 6×10^{-2} gm/m.[3]

Example 6

Source data: A practically instantaneous cross-wind line source is laid at ground level by a fast-moving vehicle, emitting material at a steady rate of 1 gm/sec m.

Meteorological data: Clear sky at night. Height of lid given as 200 m. Wind components perpendicular to the line source given as follows:

Height *metres*	Wind component *metres/second*
10	2
25	4
50	6
150	9
200	10

Problem: Measuring times from the time at which the line source is laid, calculate approximately the time at which the concentration at a point 5 km downwind of the source will reach a peak, and the interval during which the concentration at this point is likely to be greater than 1×10^{-6} gm/m^3.

Solution:

Operation	*Remarks*
1	Set cursor A on 1 on scale 2.
2*a*(i)	Diffusion category found to be Q.
2*a*(ii)	Set 5 km (*h*,*d*) line under point Q on cursor E.
2*f*	Lid is too high to be effective. Note h' (=48 m) on scale 4 under cursor A. Calculate from wind data the average wind speed from ground level up to height 48 m. This comes to about 4·0 m/sec.
23	Set cursor G on 5 km on scale 5.
24	Read time at which peak concentration is expected on scale 8 opposite speed of travel, 4·0 m/sec, on scale K. This is 21 minutes.
4*a*(ii)	Set point Q on cursor G on 5 km on (*w*,*d*) lines.
19	Set mark GG on 1×10^{-6} on scale 11.
20	Note adjustment number, 11·2, on scale 13 under cursor M.

22 Read co-ordinate, $\pm x$, on scale 5 opposite adjustment number, 11·2, on scale J. This is 460 m. This means that, when the peak concentration in the belt is at 5 km from the source, concentrations of 1×10^{-6} gm/m^3 will be at distances of about 460 m upwind and downwind of the peak. Thus the approximate interval during which the concentration at 5 km will be greater than 1×10^{-6} gm/m^3 will be 2×460 m divided by the speed of travel, 4·0 m/sec. This division is carried out with the next two operations.

23 Set cursor G on 920 m on scale 5.

24 Read required duration on scale 8 opposite 4·0 on scale K. The answer is about 4 minutes.

Thus it would be estimated that, at 5 km, concentration could be greater than 1×10^{-6} gm/m^3 from 19 to 23 minutes after the creation of the line source.

Example 7

Source data: } As in Example 6.
Meteorological data: }

Problem: Plot a graph of concentration versus time for the 5-km distance, using the wind shear method.

Solution:

Operation	*Remarks*
	Before making calculations it is advisable to plot a graph of wind speed versus height (see Figure 17) so that winds and wind shears can be easily estimated.
1, 2*a*(i), 2*a*(ii), 2*f*, 23, 24	As in Example 6.
4*a*(i)	Use wind speed graph (Figure 17) to obtain shear, i.e. difference between wind speed at ground level and that at height h'. This is about 3·8 m/sec.

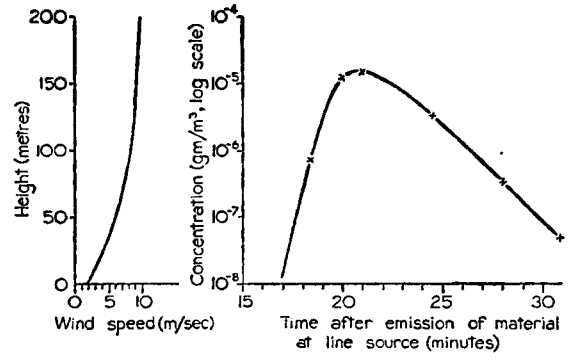

FIGURE 17. Plots of wind speed versus height and concentration versus time required in Example 7. Crosses denote concentrations 5 km downwind of line source of strength 1 gm/sec m, calculated for various times. The diffusion category is Q.

4*d*(ii) Set mean speed, 4·0 m/sec, on scale L on mark FF.

4*d*(iii) Set 5 km on scale 15 under cursor Q.

4*d*(iv) Set Δu, 3·8 m/sec, on scale O on mark II. This leaves cursor G at about 2·3 km on scale 5. This 2·3 km is the conventionally defined width of the belt.

8 Read peak concentration on scale 10 under cursor M. This is about $1{\cdot}6\times10^{-5}$ gm/m^3, and is the concentration calculated for a time 21 minutes after the emission.

Repeat operations 2*a*(ii) to 4*d*(iv) for various distances either side of 5 km, and for each distance, X, after operation 4*d*(iv) carry out the following operations:

4*	Note adjustment number on scale J opposite value of $\|5-X\|$ km on scale 5 (e.g. for $X=4$ km note adjustment number corresponding to 1 km; this comes out to be 11·0).
12	Set adjustment number on scale 13 under cursor M.
13	Read concentration on scale 11 opposite mark GG. This will be the concentration at the time read in operation 24.

The following table sets out items of calculations for various distances.

Distance, X, of peak concentration from source	Height h' over which to estimate mean speeds and wind shears	Mean speeds (approx.)	Time of travel, t, for distance X	Wind shear	Width at distance X	$\|5-X\|$	Adjustment number	Concentration at 5 km at time t
kilometres	*metres*	*m/sec*	*minutes*	*m/sec*	*kilometres*			*grams/m³*
3	30	3·3	15	2·8	1·2	2	>15	$<5\times10^{-14}$
4	37	3·6	18½	3·1	1·6	1	11·0	7×10^{-7}
4½	40	3·8	20	3·5	2·1	½	6·9	1×10^{-5}
5	45	4·0	21	3·8	2·3	0	0	2×10^{-5}
6	50	4·1	24½	4·0	2·7	1	8·8	3×10^{-6}
7	52	4·2	28	4·2	3·4	2	10·9	4×10^{-7}
8	54	4·3	31	4·4	3·9	3	11·8	4×10^{-8}

The required concentration-versus-time graph is shown in Figure 17. Note that the period during which the concentration is greater than 1×10^{-6} gm/m³ is longer than that calculated by the more approximate method in Example 6.

Example 8

Meteorological data: There is a lid on diffusion at 300 m. The mean wind velocity between ground level and 300 m is predicted to be 270 degrees 8 m/sec. The winds at ground level and at 300 m are predicted to be 240 degrees 7 m/sec and 280 degrees 10 m/sec respectively. It is stated that the mean wind direction can be predicted to within ±10 degrees and that there will be haze with weak sunshine.

Problem: How long should a cross-wind line source be to simulate the effect of an infinitely-long line source on a point 100 km downwind of the source ?

Solution:

As there appears to be sufficient wind data, the wind shear method will be used. At ground level the wind component perpendicular to the mean direction, 270 degrees, is 7 sin 30° = 3·5 m/sec. At 300 m the cross-wind component is 1·7 m/sec in the opposite direction. Therefore, the cross-wind shear between ground level and 300 m is approximately 5·2 m/sec. This shear value may be needed.

Operation	*Remarks*
2*a*(ii)	Diffusion category found to be N.
I	The distance, d_1, at which the value on scale 4 under cursor A becomes equal to the height of the lid, 300 m, is 15 km.
II	Set mark d_1 on 15 km on scale 17.
III	d_2 is read as 63 km. Therefore, the 100-km calculation will require the value of the virtual diffusion coefficient, K_ν.
IV	Set cursor F over speed of travel, 8 m/sec, on scale 14.
V	Set wind shear, 5·2 m/sec, on scale 14 under cursor F.
VI	Read K_ν on scale 15 opposite 5·2 m/sec on scale 18. This is $1{\cdot}4\times10^3$ m²/sec.
6*c*(i)	Set cursor K on 100 km on scale 7.

6*c*(ii)	Set $1{\cdot}4 \times 10^3$ on scale 15 under cursor K.
6*c*(iii)	Set 8 m/sec on scale P on mark JJ.
6*e*	Read value of width, w_y, now on scale 5 under cursor G. This is about 25 km. This is the length the line would need to be if the mean wind direction could be predicted with complete accuracy. To allow for the tolerance of ± 10 degrees add on a quantity $(2 \times 100 \times \tan 10°)$ km. This multiplication can be carried out approximately with the slide-rule by the next two operations.
6*b*	Set 20° (2×10) on scale H on mark DD. Set 100 km on scale 16 under cursor Q. Set cursor Q on mark HH.
6*e*	Read width on scale 5 under cursor G. This is about 35 km.

Thus the line source should be at least $25+35=60$ km long. Its mid-point should be in the predicted upwind direction of the point.

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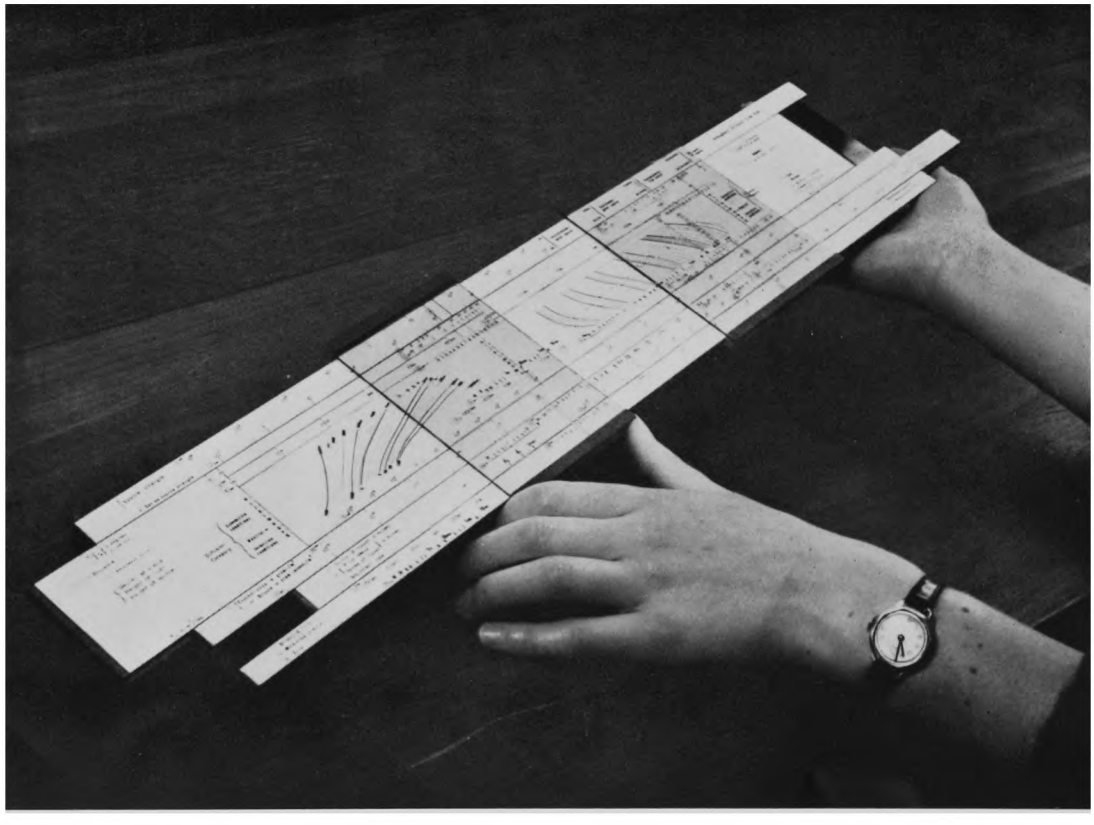

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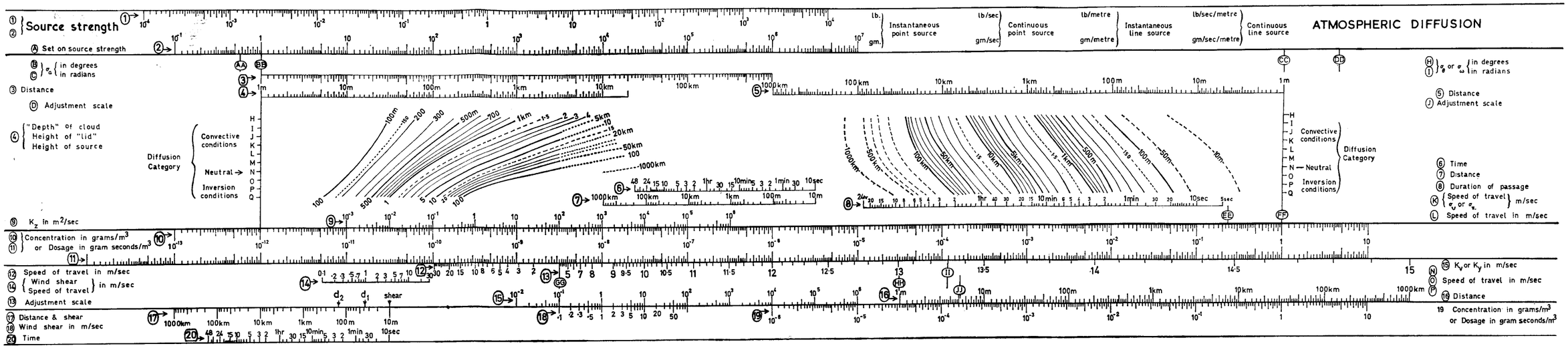
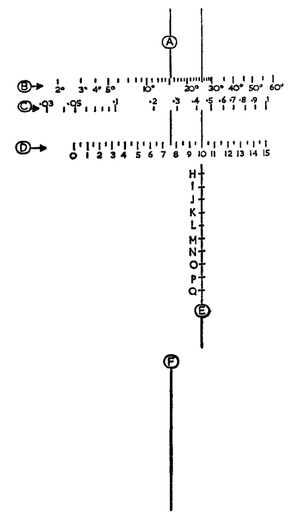
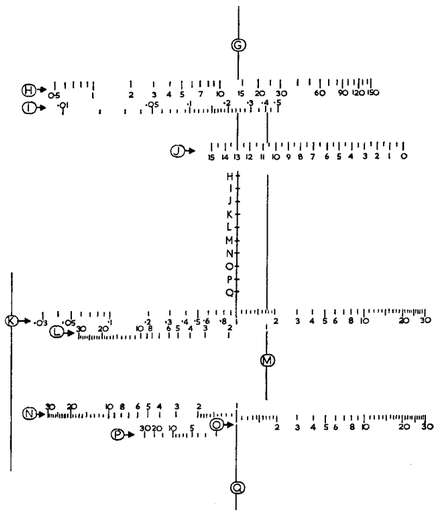
PLATE I—The atmospheric diffusion slide-rule.



(*a*) Full-scale diagram of the frame and two sliding sections of the slide-rule. The sections including scales numbered 1, 2, 10, 11, and 17–20 are fixed; the other scales are on two sliding sections.



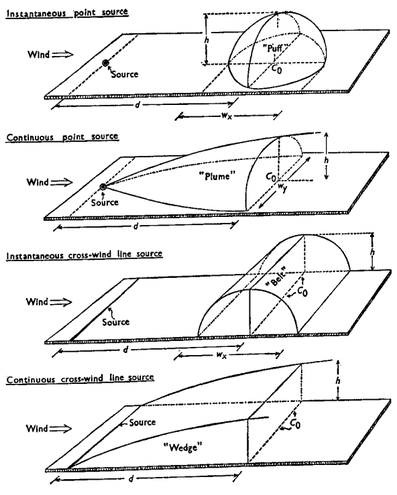
(*b*) Left-hand cursor plate.



(*c*) Right-hand cursor plate.

FIGURE 1. The atmospheric diffusion slide-rule.

When the cursors are on the frame the diffusion category letters H to Q on the upper slide are horizontally aligned with the same letters on the two cursor plates.



Diffusion Category

WIND SPEED at 10 metres above ground			DAYTIME			NIGHT		WIND FORCE AND EFFECTS	
Knots	m.p.h.	metres/sec	Strong Sunshine	Moderate Sunshine	Weak Sunshine	Overcast day or night	Thin cloud covering more than half the sky	Fairly clear, less than half the sky covered by cloud	Beaufort force	WIND EFFECTS
2	3	1	H	I	J	K	L	M	1	Wind direction shown by smoke drift, but flags hang limply
3	4	2							2	Wind felt on face Leaves rustle Flags flutter gently
4	5	3				N		O	3	Leaves and twigs move in the wind Light flags extended
5	6	4					P	Q	4	Raises dust and loose paper Small branches blown about Flags well extended
8	9	5							5	Small trees in leaf begin to sway Wavelets on inland waters
10	12	6								
15	17	8								
20	23	10								

Formulae for Peak Concentration in grams/cu metre and Dosage in gram seconds/cu metre

Peak concentration in a puff	$\frac{(1{\cdot}716)^3 Q}{h w_x w_y}$	Q = Amount of material emitted in grams or rate of emission in grams/second
Peak dosage in a puff Peak concentration in a plume	$\frac{(1{\cdot}716)^2 Q}{U h w_y}$	h = "Depth" of cloud in metres
Peak concentration in a belt	$\frac{(1{\cdot}716)^2 Q}{h w_x}$	w_x = Along-wind "width" of cloud in metres
Peak dosage in a belt Peak concentration in a wedge	$\frac{1{\cdot}716\,Q}{Uh}$	w_y = Cross-wind "width" of cloud in metres U = Wind speed in metres/second

BASIC SLIDE-RULE OPERATIONS	TYPE OF CALCULATION (C = Concentration, D = Dosage)	PLUME PUFF C_0	PLUME PUFF D_0	PLUME BELT C_0	WEDGE BELT D_0	WEDGE C_0
Basic operations	Set cursor A on source strength, Q, on scales 1 or 2	1	1	1	1	1
	Set depth, h, on scale 4 under cursor E	2	2	2	2	2
	Note adjustment number on scale D opposite height of source, h_s, on scale 4 (See note on concentrations or dosages at height c)	2*	2*	2*	2*	
	Set cursor M on mark FF					3
	Set cursor G on along-wind width, w_x, on scale 5	4		4		
	Note adjustment number on scale J opposite value of x on scale 3	4*		4*		
	Move mark FF to cursor M	5				
	Set cursor G on cross-wind width, w_y, on scale 5	6	6			
	Note adjustment number on scale J opposite value of y on scale 5	6*	6*			
	Read D_0 or C_0 now on scale 10 opposite speed of travel, u, on scale L		7		7	
	Read C_0 now under cursor M on scale 10	8		8		

METHODS OF SETTING DEPTH PARAMETERS

DATA REQUIRED	DIFFUSION CATEGORY METHOD	WIND FLUCTUATION METHOD	DIFFUSION COEFFICIENT METHOD
	State of sky and 10-metre wind	Standard deviation of wind inclination	Vertical diffusion coefficient and time or speed of travel
Operations equivalent to operation 2	2a (i) Determine diffusion category letter (ii) Set distance, d, on (h, d) lines under diffusion category letter on cursor E	2b (i) Set mark AA on source strength Q on scales 1 or 2 (ii) Set σ_θ in degrees on scale B or in radians on scale C on mark BB (iii) Set distance, d, on scale 3 under cursor E (iv) Set cursor A on source strength on scale 1 or 2	2c (i) Note: Initially the source strength must be taken as 1 on scale 2 (ii) Set cursor F on time of travel on scale 20 or Set mark d_1 on distance, d on scale 17, then set cursor F on speed of travel, u on scale 14 (iii) Set diffusion coefficient, K_z, on scale 9 under cursor E (iv) Set cursor A on 1 on scale 2
To read depth h	2e Read depth, h, now on scale 4 under cursor E		
Effect of lid	2f Check that the value on scale 4 under cursor A does not exceed height of lid, H. If it does set H on scale 4 under cursor A Note: The value on scale 4 under cursor A is referred to as h'		

Calculation of: K_u, K_v Virtual diffusion coefficients	I	Carry out operations 2a, b or c for increasing values of distance of travel, d, and note distance, d_1, at which the value on scale 4 under cursor A becomes equal to height of lid, H
d_2 = Distance beyond which the virtual coefficients are used	II	Set mark d_1 on the value of d_1 on scale 17
	III	Read value of d_2 now on scale 17 opposite mark d_2
	IV	Set cursor F over u (speed of travel) on scale 14
s_x s_y Limiting shear almost attained at distance $2d_2$	V	Set Δu (wind shear over depth H) on scale 14 under cursor F
	VI	Read K_u on scale 15 opposite Δu on scale 18
	VII	Read limiting shear, s, on scale 17 opposite mark "shear"
	VIII	Cross-wind values, K_v and s_y, can be found by similar operations using Δv in place of Δu

	DIFFUSION CATEGORY METHOD	WIND FLUCTUATION METHOD	DIFFUSION COEFFICIENT METHOD	WIND SHEAR METHOD
Data required	State of sky and 10-metre wind	For w_x: Standard deviation, σ_θ, of wind in x direction and speed of travel, u For w_y: Standard deviation, σ_θ, of wind in y direction and speed of travel, u or Standard deviation, σ_α, of wind direction or Range over which wind direction fluctuates	Speed or time of travel of cloud and horizontal diffusion coefficients K_x for w_x K_y for w_y	Speed of travel of cloud and wind speed components up to a height to be noted in 4 d (i) u components for w_x v components for w_y
Operations equivalent to operation 4	4a (i) Determine diffusion category (ii) Set distance d on (w, d)lines under diffusion category letter (But see note on puff type calculations for short distances)	4b (i) Set σ_θ on scale K on EE (ii) Set distance, d, on scale 16 under cursor (iii) Set speed of travel, u, on scale N on mark HH	4c (i) Set cursor K on time, t, on scale 6 or on distance, d, on scale 7 (ii) Set K_x on scale 15 under cursor K (iii) If time, t, is being used set cursor Q on mark JJ: If speed, u, is being used set u on scale P on mark JJ.	4d If $d < d_2$ (i) From wind data obtain wind shear Δu over depth h' (ii) Set u on scale L on mark FF (iii) Set d on scale 15 under cursor Q (iv) Set Δu on scale O on mark II If $d > d_2$ Apply operations 4c with virtual diffusion coefficient, K_U
Read width if required	4e Read value of width, w_x now on scale 5 under cursor line G			
Operations equivalent to operation 6 in Table I	6a same as operations 4a	6b If σ_θ is given: Similar to operations 4b with σ_θ in place of σ_{-u}	6c Similar to operations 4c with K_y in place of K_x	6d Similar to operations 4d with Δv in place of Δu, and K_V in place of K_U
	If σ_θ or θ_w is given: (i) Set σ_θ in degrees on scale H or in radians on scale I over mark CC, or set θ_w in degrees on scale H over mark DD (ii) Set d on scale 16 under cursor line Q (iii) Move right-hand cursor to set cursor line Q on mark HH			
Read width if required	6e Read value of width, w_y now on scale 5 under cursor line G			

(*d*) Summary of slide-rule operations and notes on the back of the slide-rule.

FIGURE 1. The atmospheric diffusion slide-rule.

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